**Additional File 2:** Expressing “pseudo” model data in terms of spatial and temporal correlations () and variance ratios ()

Based on papers by Butland et al,[1] and Reeves et al,[2] we can incorporate both classical and Berkson error into “true” daily air pollution data ( to obtain pseudo modelled data ( for each site using the following formula:

In order to incorporate error at both spatial (and temporal levels (we can expand this formula using a generalisation of second-order regression as outlined in Cox and Hinkley,[3] as follows:

We want to express equation (2.2) in terms of:

a) The temporal correlation between true and model data within site=; b) the spatial correlation between true and model 5-year means=; c) the ratio of model to true variances temporally (variances of daily data within site) =; and d) the ratio of model to true variances spatially (variances of 5-year means across sites) =.

Similarly

Using (2.3) and (2.4) we can re-write (2.2)

In our simulations we assume that the variation across time is the same within each site i.e. , for all .

**References**

[1] Butland BK, Armstrong B, Atkinson RW, Wilkinson P, Heal MR, Doherty RM, Vieno M. Measurement error in time-series analysis: a simulation study comparing modelled and monitored data. BMC Med Res Methodol. 2013;13:136.

[2] Reeves GK, Cox DR, Darby SC, Whitley E. Some aspects of measurement error in explanatory variables for continuous and binary regression models. Statist Med. 1998;17:2157-77.

[3] Cox DR, Hinkley DV: Appendix 3 Second-order regression for arbitrary random variables. In Theoretical Statistics. London: Chapman and Hall; 1974. p. 475-477.